Fórmulas de reducción trigonométricas

$$I_n = \int sen^n u \, du = -\frac{1}{n} sen^{(n-1)} u \cos u + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \cos^n u \, du = \frac{1}{n} \cos^{(n-1)} u \; sen\, u + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \sec^n u \, du = \frac{1}{n-1} \sec^{(n-2)} u \tan u + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \csc^n u \, du = -\frac{1}{n-1} \csc^{(n-2)} u \cot u + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - I_{n-2}$$

Ejemplo: ($n=2$; $n-2=0$)

$$I_{(2)} = \int sen^2 \theta \, d\theta = -\frac{1}{2} sen^{(1)} \theta \cos \theta + \frac{1}{2} \int d\theta = -\frac{1}{2} sen\,\theta \cos \theta + \frac{\theta}{2} + C$$

$$= \frac{\theta}{2} - \frac{sen\,\theta \cos\theta}{2} + C = \frac{2\theta - 2\, sen\,\theta \cos \theta}{4} + C = \frac{2\theta - sen(2\theta)}{4} + C$$

Fórmulas de cuadrados trigonométricos

$$\int sen^2 u \, du = \frac{2u - sen(2u)}{4} + C \qquad \int \cos^2 u \, du = \frac{\cos u \; sen\, u + u}{2} + C$$

$$\int \tan^2 u \, du = \tan u - u + C \qquad \int \cot^2 u \, du = -\cot u - u + C$$

$$\int \sec^2 u \, du = \tan u + C \qquad \int \csc^2 u \, du = -\cot u + C$$

Identidades Hiperbólicas

Las funciones hiperbólicas son análogas a las trigonométricas, pero se basan en la curva hipérbola en lugar del círculo.

$$senh\, x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

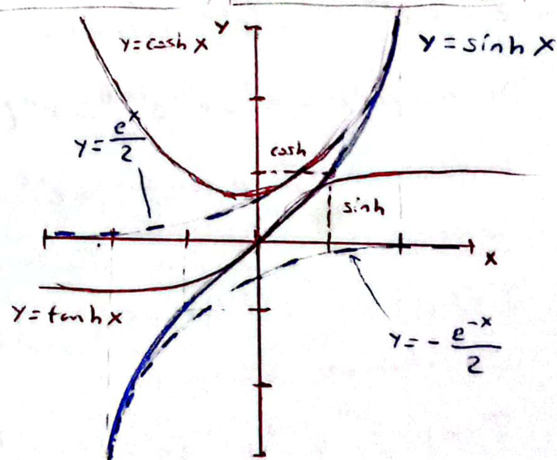
$$\coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \qquad sech\, x = \frac{1}{\cosh x} \qquad csch\, x = \frac{1}{senh\, x}$$

$$\cosh^2 x - senh^2 x = 1 \qquad senh\, 2x = 2\, senh\, x \cosh x \qquad \cosh 2x = \cosh^2 x + senh^2 x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2} \qquad senh^2 x = \frac{\cosh 2x - 1}{2} \qquad \tanh^2 x = 1 - sech^2 x \qquad \coth^2 x = 1 + csch^2 x$$

Derivadas e integrales (7.7 p.428)

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$senh\, x$	$\cosh x$	$\cosh x + C$
$\cosh x$	$senh\, x$	$senh\, x + C$
$\tanh x$	$sech^2 x$	$\ln(\cosh x) + C$
$\coth x$	$-csch^2 x$	$\ln(senh\|x\|) + C$
$sech\, x$	$-sech\, x \tanh x$	$\arctan(senh\, x) + C$
$csch\, x$	$-csch\, x \coth x$	$\ln(\tanh \frac{\|x\|}{2}) + C$
$senh^2 x$	$2 \cosh x \sinh x$	$\frac{\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} - 2x}{4} + C$
$\cos^2 x$	$2 \cosh x \sinh x$	$\frac{\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x}{4} + C$
$\tan^2 x$	$2\, sech^2 x \tanh x$	$x - \frac{2}{e^{-2x} + 1} + C$



Número e: $e \simeq \left(1+\frac{1}{n}\right)^n$ Ejemplo: $1'001^{1000} = \left(1+\frac{1}{1000}\right)^{1000} \approx e$

$a^n = e^{n \ln a} \quad \forall a, n \in \mathbb{R}$

Identidades trigonométricas

- $\text{sen}^2 x + \cos^2 x = 1$ - $\sec^2 x - \tan^2 x = 1$ - $\csc^2 x - \cot^2 x = 1$

- $\text{sen } 2x = 2 \text{ sen} x \cos x$ - $\cos 2x = \cos^2 x - \text{sen}^2 x = 2\cos^2 x - 1 = 1 - 2\text{sen}^2 x$
- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

- $\text{sen}^2 x = \frac{1-\cos 2x}{2}$ - $\cos^2 x = \frac{1+\cos 2x}{2}$ - $\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$

- $\text{sen}(u \pm v) = \text{sen } u \cos v \pm \cos u \text{ sen } v$ - $\cos(u \pm v) = \cos u \cos v \mp \text{sen } u \text{ sen } v$
- $\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

Integrales

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ (arctan)

$\int \frac{dx}{\sqrt{a^2+x^2}} = \text{senh}^{-1}\frac{x}{a} + C$ (arcsinh)

$\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \text{sen}^{-1}\frac{x}{a} + C$ (arcsin)

$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$ (arcsec)

$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a} + C$ (arccosh)

$\int x^n e^x\,dx = x^n e^x - n x^{n-1} e^x + n(n-1)x^{n-2}e^x - \dots + n!\,x e^x - n!\,e^x + C$

Números complejos: Euler

- $z = re^{i\theta}$ - $e^{i\theta} = (\cos\theta + i\sin\theta)$ - $z = r(\cos\theta + i\sin\theta)$

De Moivre's: - $z^n = r^n e^{in\theta} = r^n[\cos(n\theta) + i\sin(n\theta)]$

Conversiones: - $a\cos\theta = ae^{i\theta}$ - $a\sin\theta = -ae^{i\theta}$

Funciones

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

Transformada de Laplace

$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)\,e^{i\alpha x}\,dx = F(\alpha)$

Transformada de Fourier

$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}\,dx = (n-1)!$

Función gamma

Funciones Hiperbólicas Inversas

$\operatorname{sech}^{-1} x = \cosh^{-1} \frac{1}{x}$ | $\operatorname{csch}^{-1} x = \sinh^{-1} \frac{1}{x}$ | $\coth^{-1} x = \tanh^{-1} \frac{1}{x}$

$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ $\quad -\infty < x < \infty$

$\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$ $\quad x \geq 1$

$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$ $\quad |x| < 1$

$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ $\quad 0 < x \leq 1$

$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right)$ $\quad x \neq 0$

$\coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1}$ $\quad |x| > 1$

Derivadas e integrales de las funciones hiperbólicas inversas

$f(u)$	$\frac{df}{du}$
$\operatorname{senh}^{-1} u$	$\frac{1}{\sqrt{1+u^2}}\,du$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}\,du$
$\tanh^{-1} u$	$\frac{1}{1-u^2}\,du$
$\coth^{-1} u$	$\frac{1}{1-u^2}\,du$
$\operatorname{sech}^{-1} u$	$\frac{1}{u\sqrt{1-u^2}}\,du$
$\operatorname{csch}^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\,du$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \frac{u}{a} + C & \text{si } |u| < 1 \\ \frac{1}{a} \coth^{-1} \frac{u}{a} + C & \text{si } |u| > 1 \end{cases} \quad *$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \left|\frac{u}{a}\right| + C$$

Ejemplo!

$$\int \frac{2\,dx}{\sqrt{3+4x^2}}$$

$u^2 = 4x^2 \Rightarrow u = 2x$ y $du = 2$. $a^2 = 3 \Rightarrow a = \sqrt{3}$

$$\int \frac{2\,dx}{\sqrt{3+4x^2}} = \int \frac{du}{\sqrt{a^2+u^2}} = \operatorname{senh}^{-1} \frac{u}{a} + C = \boxed{\operatorname{senh}^{-1} \frac{2x}{\sqrt{3}} + C}$$

* Ejemplo: $\int_{\frac{5}{4}}^{2} \frac{dx}{1-x^2}$! sea $a=1$ y $u=x$, $x > 1 \ \forall x \in [\frac{5}{4}, 2]$. Por tanto,

$$\int_{\frac{5}{4}}^{2} \frac{dx}{1-x^2} = \coth^{-1} x \Big]_{\frac{5}{4}}^{2} = \coth^{-1} 2 - \coth^{-1} \frac{5}{4} = \ln\sqrt{3} - \ln 3 = \ln \frac{\sqrt{3}}{3}$$

Usando el arcotangente hiperbólico:

$$\int_{\frac{5}{4}}^{2} \frac{dx}{1-x^2} = \tanh^{-1} x \Big]_{\frac{5}{4}}^{2} = \tanh^{-1} 2 - \tanh^{-1} \frac{5}{4}$$

Pero $\tanh^{-1} 2 \ \frac{1}{2}\ln \frac{1+2}{1-2} = \frac{1}{2} \ln -\frac{3}{1}$. Y $\ln(-3)$ no está definido.

- Si el intervalo de la integral contiene 1, la integral es divergente.

Sustitución trigonométrica

$$\int_0^{\ln 4} \frac{e^t\,dt}{\sqrt{e^{2t}+9}}$$

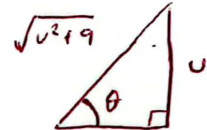
- Se computa la antiderivada. Se usa la sustitución $u^2=e^{2t}$, $u=e^t$, y $du=e^t$.

$$\int \frac{e^t\,dt}{\sqrt{e^{2t}+9}} = \int \frac{du}{\sqrt{u^2+9}} \leftarrow e^t = du$$

- Dado que u^2 y 9 son cuadrados perfectos, se usa una sustitución trigonométrica. Sea $a=3$, $u=3\tan\theta$, $du=3\sec^2\theta$ y $\theta=\arctan\frac{u}{3}$!

$$\int \frac{du}{\sqrt{u^2+9}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9\tan^2\theta+9}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9(\tan^2\theta+1)}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9\sec^2\theta}} = \int \frac{3\sec^2\theta\,d\theta}{3\sec\theta} = \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta+\tan\theta| + C$$



$a=3$ $(a^2=9)$

$u=a\tan\theta=3\tan\theta$

$du=3\sec^2\theta$

$\sec\theta=\frac{\sqrt{u^2+9}}{3}$

$\tan\theta=\frac{u}{3}$

$\theta=\arctan\left(\frac{u}{3}\right)$

- Se deshacen las sustituciones:

$$\ln|\sec\theta+\tan\theta|+C = \ln\left|\frac{\sqrt{u^2+9}}{3}+\frac{u}{3}\right|+C = \boxed{\ln\left|\frac{\sqrt{e^{2t}+9}+e^t}{3}\right|+C}$$

- Se computa la integral usando la antiderivada

$$\int_0^{\ln 4} \frac{e^t\,dt}{\sqrt{e^{2t}+9}} = \ln\left|\frac{\sqrt{e^{2t}+9}+e^t}{3}\right|\Bigg]_0^{\ln 4} = \ln\left(\frac{\sqrt{25}+4}{3}\right)-\ln\left(\frac{\sqrt{10}+1}{3}\right)$$

$e^{2\ln 4}=4^2=16$, $e^{\ln 4}=4$

$$= \ln\left(\frac{9}{3}\right)-\ln\left(\frac{\sqrt{10}+1}{3}\right) = \ln 9 - \ln 3 - (\ln(\sqrt{10}+1) - \ln 3) = \boxed{\ln 9 - \ln(\sqrt{10}+1)} \quad 0'771162138$$

$$\int \frac{x^2+1}{x^4+1}\,dx = \int \frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(x^2+\frac{1}{x^2}\right)}\,dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

- Se usa la sustitución $u=x-\frac{1}{x}$, de modo que $u^2=\left(x-\frac{1}{x}\right)^2=\left(x^2+\frac{1}{x^2}\right)-2$, y $du=1+\frac{1}{x^2}$. Y $a=\sqrt{2}$ $(a^2=2)$.

$$\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx = \int \frac{du}{u^2+2} = \frac{1}{\sqrt{2}}\arctan\left(\frac{u}{\sqrt{2}}\right) = \boxed{\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}-\frac{1}{x\sqrt{2}}\right)+C}$$

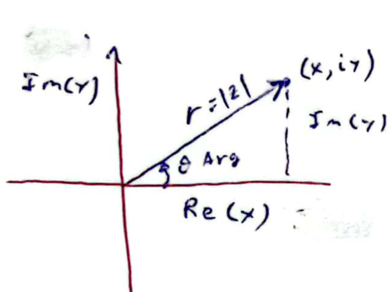
si $u^2=x^2+\frac{1}{x^2}-2$, entonces $u^2+2=x^2+\frac{1}{x^2}$.

Complex number $i = \sqrt{-1}$

$Re + i\,Im$ or $Re + j\,Im$ $\quad (5+8i) \to Re = x = 5,\ Im = y = 8$

$x = Re \quad y = Im$

Argand Diagram



$\sin\theta = \frac{y}{r} = \frac{Im}{r} \quad y = r\sin\theta$

$\cos\theta = \frac{x}{r} = \frac{Re}{r} \quad x = r\cos\theta$

$Arg(z) = \theta = \arcsin\frac{y}{r} = \arcsin\frac{Im}{r} = \arccos\frac{x}{r} = \arccos\frac{Re}{r}$

Euler formula

$z = re^{i\theta}$ $\quad z^n = r^n e^{in\theta} = r^n(\cos(n\theta) + i\sin(n\theta))$ De Moivre's

$e^{i\theta} = (\cos\theta + i\sin\theta)$

$z = r(\cos\theta + i\sin\theta)$

Equality

$a + bi = c + di \Leftrightarrow a = c,\ b = d$

Sum $(a \pm bi) \pm (c \pm di) = (a \pm c) \pm (b \pm d)i$

Product $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Quotient $\frac{a+bi}{c+di} = \frac{ac+bd}{a^2+b^2} - \frac{ad-bc}{a^2+b^2}i \quad \frac{a+bi}{c+di}\frac{(c-di)}{(c-di)}$

Power $z^n = r^n e^{in\theta}$

Conversions

A: Constant

$i = \sqrt{-1}$ $\quad i^2 = -1$ $\quad \sqrt{-4} = \sqrt{4}\sqrt{-1} = 2i$ $\quad A\cos(\theta) = Ae^{i\theta}$ $\quad A\sin(\theta) = -Ae^{i\theta}$

$z = -7 + 24i \quad |z| = \sqrt{49+576} = 25$

$z = -25(\cos\theta - i\sin\theta) \quad \arg(z) = \arcsin\frac{24}{25} = 1'287002$

using $z = re^{i\theta}$

$r = e^{i\theta} + e^{-i\theta}$

Equation

$(3+4i)^2 - 2(x - iy) = x + iy$

$(3+4i)^2 = (3+4i)(3+4i) = (9-16) + (12+12)i = (-7 + 24i)$

$-7 + 24i - 2x + 2iy = x + iy \to x = Re,\ y = Im$

$x = -7 - 2x \quad y = 24 - 2y$

$x = -\frac{7}{3} \quad y = -24$

Example

$$x = \frac{5 \pm \sqrt{-4}}{2} \rightarrow x = \frac{5 \pm i\sqrt{4}}{2} \rightarrow x = \frac{5 \pm 2i}{2} = \frac{1}{2}(5 \pm 2i) = \left(\frac{5}{2} \pm i\right)$$

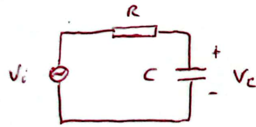
$$z = \frac{5}{2} + i \quad \bar{z} = \frac{5}{2} - i \rightarrow |z| = \sqrt{\left(\frac{5}{2}\right)^2 + 1^2} = \sqrt{\frac{25}{4} + \frac{4}{4}} = \sqrt{\frac{29}{4}} = \frac{1}{2}\sqrt{29}$$

$$r = |z| = \frac{1}{2}\sqrt{29} \rightarrow z = re^{i\theta} \rightarrow e^{i\theta} = \cos\theta + i\sin\theta \rightarrow z = \frac{1}{2}\sqrt{29}(\cos\theta + i\sin\theta)$$

$$\arg(z) = \theta \rightarrow \theta = \arcsin\frac{y}{r} = \arccos\frac{x}{r} = \arctan\frac{y}{r} \rightarrow \theta = \arctan\frac{2}{5}$$

$$z = \frac{1}{2}\sqrt{29}\left(\cos\left(\arctan\frac{2}{5}\right) + i\sin\left(\arctan\frac{2}{5}\right)\right) \quad \Big| \quad z = \frac{1}{2}\sqrt{29}\, e^{i\arctan\left(\frac{2}{5}\right)}$$

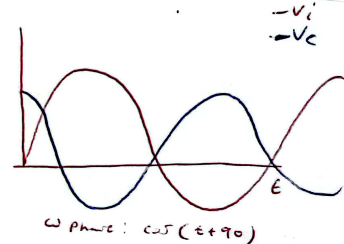
$$\bar{z} = \frac{1}{2}\sqrt{29}\left(\cos\left(-\arctan\frac{2}{5}\right)\right) + i\sin\left(-\arctan\frac{2}{5}\right)\Big) \quad \Big| \quad \bar{z} = \frac{1}{2}\sqrt{29}\, e^{-i\arctan\left(\frac{2}{5}\right)}$$



$$V_i = V_c + RC\frac{dV_c}{dt} \qquad \frac{dV_c}{dt} = j\omega V_c$$

$$V_c = A\cos(\omega t + \theta)$$

$$V_c = A e^{j(\omega t + \theta)} \qquad A\cos(\theta) = A e^{j(\theta)}$$



Find V_c

A: Some constant (Real)
j: $\sqrt{-1}$ (complex part)
ω: Phase

$V_i = V_c + RC(j\omega V_c)$ Subst. $\frac{dV_c}{dt}$

$V_i = V_c(1 + j\omega RC)$ Factor V_c

$V_c = \frac{1}{1 + j\omega RC} V_i$ Divide both by $1 + j\omega RC$

ω phase: $\cos(t + 90)$

Capacitor eq.

$$C\frac{dV}{dt} = I \qquad V = A e^{j(\omega t + \theta)} \quad V = A\cos(\omega t + \theta) \qquad I = j\omega C \underbrace{A e^{j(\omega t + \theta)}}_{V} \qquad I = j\omega C V \qquad \frac{V}{I} = \frac{1}{j\omega C}$$

(capacitor impedance)

Actual voltage is the real component.

Asuming $\omega = R = c = 1$

$$V_c = \frac{1}{1+j} V_i \rightarrow V_c = \left(\frac{1}{2} - \frac{1}{2}j\right)V_i \rightarrow V_c = \frac{\sqrt{2}}{2} e^{j(-45°)} V_i$$

$$V_c = \frac{\sqrt{2}}{2} e^{j(-45°)} \cdot 3e^{j(t+10°)} = \frac{3\sqrt{2}}{2} e^{j(t-35°)} = \frac{3\sqrt{2}}{2}\cos(t - 35°)$$ (Ignoring the imaginary part)

$$I = -\omega C A \sin(\omega t + \theta) = j\omega C A e^{j(\omega t + \theta)} = j\omega C V$$

Descomposición parcial de fracciones

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$$

1. Sea $x-r$ un factor linear de $g(x)$. Supon que $(x-r)^m$ es la mayor potencia de $x-r$ que divide $g(x)$. Entonces, a éste factor, asignale la suma de las m fracciones parciales:

$$\frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + \dots + \frac{A_m}{(x-r)^m}$$

Haz ésto para los distintos factores lineales de $g(x)$.

$$\boxed{x^2+4x+1 = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3}}$$

2. Sea x^2+px+q un factor irreducible cuadrático de $g(x)$ tal que x^2+px+q no tenga una raíz real. Supon que $(x^2+px+q)^n$ es la máxima potencia de éste factor que divide $g(x)$. A éste factor, asignale la suma de las n fracciones parciales:

$$\frac{B_1x+C_1}{(x^2+px+q)} + \frac{B_2x+C_2}{(x^2+px+q)^2} + \dots + \frac{B_nx+C_n}{(x^2+px+q)^n}$$

Haz ésto para los factores cuadráticos de $g(x)$.

$$A(x+1)(x+3) + B(x-1)(x+3) + C(x-1)(x+1) =$$
$$= Ax^2+4Ax+3A + Bx^2+2Bx-3B + Cx^2 - C =$$
$$= (A+B+C)(x^2) + (4A+2B)(x) + (3A-3B-C)$$

3. Establece la fracción original $\frac{f(x)}{g(x)}$ igual a la suma de éstas fracciones parciales. Despeja la ecuación resultante y reordena los términos en orden decreciente de las potencias de x.

4. Iguala los coeficientes de las correspondientes potencias de x y resuelve la ecuación resultante para los coeficientes indeterminados.

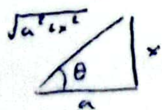
$A+B+C = 1$ coef. x^2
$4A-2B+0C = 4$ coef. x^1
$3A-3B-C = 1$ coef. x^0

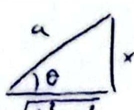
$$\text{rref}\left(\begin{bmatrix}1&1&1&1\\4&2&0&4\\3&-3&-1&1\end{bmatrix}\right) = \begin{bmatrix}1&0&0&\frac{3}{4}\\0&1&0&\frac{1}{2}\\0&0&1&-\frac{1}{4}\end{bmatrix}\begin{bmatrix}x^2\\x^1\\x^0\end{bmatrix}$$

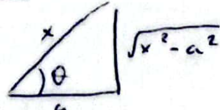
$A = \frac{3}{4}$, $B = \frac{1}{2}$, $C = -\frac{1}{4}$

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx = \int\left[\frac{3}{4}\frac{1}{x-1} + \frac{1}{2}\frac{1}{x+1} + -\frac{1}{4}\frac{1}{x+3}\right]dx = \boxed{\frac{3}{4}\ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x+3| + C}$$

Sustitución trigonométrica







$x = a\tan\theta$ $\quad$ $x = a\sin\theta$ $\quad$ $x = a\sec\theta$

$\sqrt{a^2+x^2} = a|\sec\theta|$ $\quad$ $\sqrt{a^2-x^2} = a|\cos\theta|$ $\quad$ $\sqrt{x^2-a^2} = a|\tan\theta|$

$$\int\frac{dx}{\sqrt{4+x^2}} = \int\frac{2\sec^2\theta\,d\theta}{\sqrt{4\sec^2\theta}} = \int\frac{\sec^2\theta\,d\theta}{|\sec\theta|} = \int\sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C = \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$

$a = 2$

$x = 2\tan\theta$ $\quad$ $dx = 2\sec^2\theta\,d\theta$ $\quad$ $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$4+x^2 = 4+4\tan^2\theta = 4(1+\tan^2\theta) = 4\sec^2\theta$

Integración

$$\int_0^{\pi} e^{\cos\theta} \sin 2\theta \, d\theta = 2\int e^{\cos\theta} \sin\theta \cos\theta \, d\theta \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

Sea $u = \cos\theta$, $du = -\sin\theta \, d\theta$, $-du = \sin\theta \, d\theta$

$$-2\int e^{u} \cdot u \, du$$

Es necesario una integración por partes. Cambiamos u por t y recalculamos los límites de la integración en base a t.

$t = u = \cos\theta$. $t(a) = t(0) = \cos 0 = 1$, $t(b) = t(\pi) = \cos\pi = -1$.

$$-2\int_1^{-1} e^{t} \cdot t \, dt = 2\int_{-1}^{1} e^{t} \cdot t \, dt$$

Siempre que haya un polinomio, u es el polinomio.

Sea $u = t$, $du = dt$, y sea $dv = e^t$, $v = \int e^t dt = e^t$:

$$2\left[\left[te^t\right]_{-1}^{1} - \int e^t dt \right] = 2\left[te^t - e^t\right]_{-1}^{1} = 2\left[(1e^1 - e^1) - (-1e^{-1} - e^{-1})\right] =$$

$$= 2\left(\frac{1}{e} + \frac{1}{e}\right) = \boxed{\frac{4}{e}}$$

Movimiento ideal de proyectil

$$\boxed{r = (v_0 \cos\alpha)t\,i + \left((v_0 \sin\alpha)t - \frac{1}{2}gt^2\right)j}$$

$$r = (x_0 + (v_0 \cos\alpha)t)i + (y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2)j$$

$$(y_0 + (v_0 \sin\alpha)t \pm \frac{1}{2}gt^2)j$$

- r: Función vectorial
- V_0: Velocidad inicial (m/s)
- t: Parámetro de tiempo (s)
- g: Constante de gravedad (En la tierra $\approx 9'8$ m/s)

Altura máxima: $\dfrac{(v_0 \sin\alpha)^2}{2g}$

Tiempo de vuelo: $\dfrac{2V_0 \sin\alpha}{g}$

Rango: $\dfrac{v_0^2}{g} \sin 2\alpha$